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SITE-INDEX AS A FUNCTION OF SOIL AND CLIMATIC CHARACTERISTICS (A PRELIMINARY NOTE BASED ON MAN-MADE STANDS OF *PINUS RADIATA* D. DON. IN NEW SOUTH WALES, AUSTRALIA)*

An equation expressing site-index as a function of soil and climatic characteristics is given. Data were collected with standardized methods from pure, even-aged (20 years) plantations of *Pinus radiata* D. Don. in New South Wales, Australia.

Although the view is generally accepted that the vegetation of an area, through its physiognomy and productivity, provides a tangible integration of the total site complex, there frequently exists a distrust of efforts to express quantitatively the relationship between site productivity and the soil and climatic characteristics of the site.

No doubt deciphering the structure of an equation expressing site index in terms of environmental variables appears to be a formidable task. The determination of the coefficients and exponents of the variables is difficult but of necessity this is brought about by the complexity of the relationship and the number of factors that participate in the phenomenon. However, the basic difficulty in making investigations of this kind lies in the lack of a sufficient number

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of empirical data, collected by standardized methods. The International Biological Programme has initiated standardized data gathering action within its framework, to test further and more vigorously the application of mathematical empirical methods to ecological data.

The Australian co-authors (F. R. Humphreys and S. W. Gentle) had data of this type collected from pure, even-aged stands of *Pinus radiata* D. Don. growing in a number of man-made plantations in New South Wales. Data included climatological characteristics, among them monthly records of precipitation, temperature extremes, and atmospheric humidity. A site index term was derived from mean height of the dominant trees at twenty years of age. Soil chemical data concerned total P, exchangeable Ca, Mg, K, Na, and Al, from two depths – 0–7.5 cm and 30–40 cm – and soil physical data provided the percentage of soil particles > 2 mm. In addition, for the upper layer of soil, sieve analysis was also employed (five fractions from 2 mm up to 0.06 mm).

Following the ideas used by Dahl, Selmer-Anderssen and Saether (1961) in reviewing the data of Viro (1955), Humphreys (1964) applied the notion of soil/stone ratio, termed in this paper „ratio of upright soil", defined as follows:

$$\text{Ratio of upright soil } (\Omega) = \frac{\text{Volume of soil sample} - \text{Volume of sample} > 2 \text{ mm}}{\text{Volume of soil sample}}$$

It was found to be an important factor in explaining productivity where soils are appreciably stony.

Preliminary mathematical analysis of the data revealed that the chemical characteristics of the 0–7.5 cm level were very much more important in explaining the site index than were the chemical features of the 30–40 cm level. Conversely the „upright soil ratio" of the deeper level was slightly more important than in the upper level. Statistical elaboration of this data by computer indicated that „effective values" would be the most promising way to bring this data into the overall model.

Using the general equations:

$$\tilde{\Omega} = \alpha_o \Omega_A + (1 - \alpha_o)\Omega_B \tag{1}$$

$$\tilde{E} = \beta_o E_A + (1 - \beta_o)E_B \tag{2}$$

where: $\tilde{\Omega}$ – effective value of upright soil ratio, $\tilde{E}$ – effective value of each separate soil chemical element, A – subscript indicating soil depth of 0–7.5 cm, B – subscript indicating – soil depth of 30–40 cm – substitution of data from three

plantation localities, Green Hills, Belanglo and Lidsdale gives:

$$\tilde{\Omega} = 0.42\,\Omega_A + 0.58\,\Omega_B \tag{3}$$

$$\tilde{E} = 0.8\,E_A + 0.2\,E_B \tag{4}$$

Critical soil and climatic factors, elucidated from the preliminary mathematical evaluation, are listed in Tables I and II, respectively. A humidity index q has been derived from the formula:

$$q = \frac{0.043}{12} \sum_{x=\mathrm{I}}^{\mathrm{XII}} \frac{R_x}{p'_x \left(1 - \frac{\ddot{\imath}_x}{100}\right)} \tag{5}$$

where: R – precipitation in mm, p' – mean saturated water vapour pressure in mm Hg, x – subscript indicating month of year, $\ddot{\imath}$ – mean atmospheric relative humidity in per cent.

Climatological characteristics

Tab. I

Locality, latitude, elevation	Mean annual values*					
	precipitation (R) (mm)	temperature (t) (° C)	relative humidity ($\ddot{\imath}$) (%)	humidity index (q)	$1 - e^{-q}$	$p'\left(1 - \frac{\ddot{\imath}}{100}\right)$
Green Hills 35°27′ S 2700 feet	1184	11.4	63	1.44	0.763	3.95
Belanglo 34°34′ S 2200 feet	978	13.4	68	1.15	0.685	3.74
Lidsdale 33°26′ S 3200 feet	831	12.0	68	1.08	0.660	3.59

*e – base of natural logarithms; further explanations of signs – see text.

It must be emphasized that the duration of the active growth season for *Pinus radiata* in these localities is effectively 365 days, with no true dormant season.

Having collected and transformed the data (Tab. I, II), the objective became

to express the site index as a predictive equation in these terms. In considering the structure of a possible hypothetical model that would be capable of expressing the relationship, two equations were used as starting points. The Mitscherlich equation:

Mean stand and soil characteristics

Tab. II

Locality	Number of plots (n)	Stand height (age 20 years – H_{20}) (feet)	Upright soil ratio ($\widetilde{\Omega}$)	Mean size of soil particles (d) * (mm)	Chemical characteristics		
					$\widetilde{P}$ total (ppm)	$\widetilde{Ca}$ exchangeable (me, %)	$\widetilde{Mg}$ exchangeable (me, %)
Green Hills	14	90	0.95	0.188	210	4.50	1.26
Belanglo	12	66	0.81	0.178	120	0.73	0.56
Lidsdale	27	67	0.73	0.156	110	0.55	0.56

*Computed by Kozeny method (see Czarnowski 1964, p. 34).

$$y = y_{\max} \prod_{x=1}^{n} \left(1 - e^{-cE_x}\right)\left(e^{-bE_x^2}\right) \tag{6}$$

in which E concerns chemical properties and y means annual growth in biomass per surface unit, served for part of the data; for the remaining site characteristics the Czarnowski frame equation (Czarnowski 1964) seemed a useful starting point.

This equation is:

$$\acute{N} = \check{C} \cdot p \left(1 - \frac{\ddot{i}}{100}\right) \cdot \frac{(1 - e^{-q})}{d \cdot [C_{\square} (w - W_F)^2 + 1]} \sqrt{1 + \gamma \acute{N}_{sp}} \tag{7}$$

where: $\check{C}$, $C_{\square}$, W_F, γ are constants for a given species, $\acute{N}$ – site indicator almost directly proportional to the traditional H_{20} (Czarnowski 1961), w – quantification of site humidity:

$$w = C_4 \frac{(1 - e^{-q})}{d^2} \cdot \frac{(1 - \sigma)^2}{\sigma^3} \tag{8}$$

where: σ – soil porosity (no data available, so $\widetilde{\Omega}$ or its functions were substituted), C_4 – proportionality constant; other symbols are as in Tables I and II.

The method of successive approximations has been used by the senior

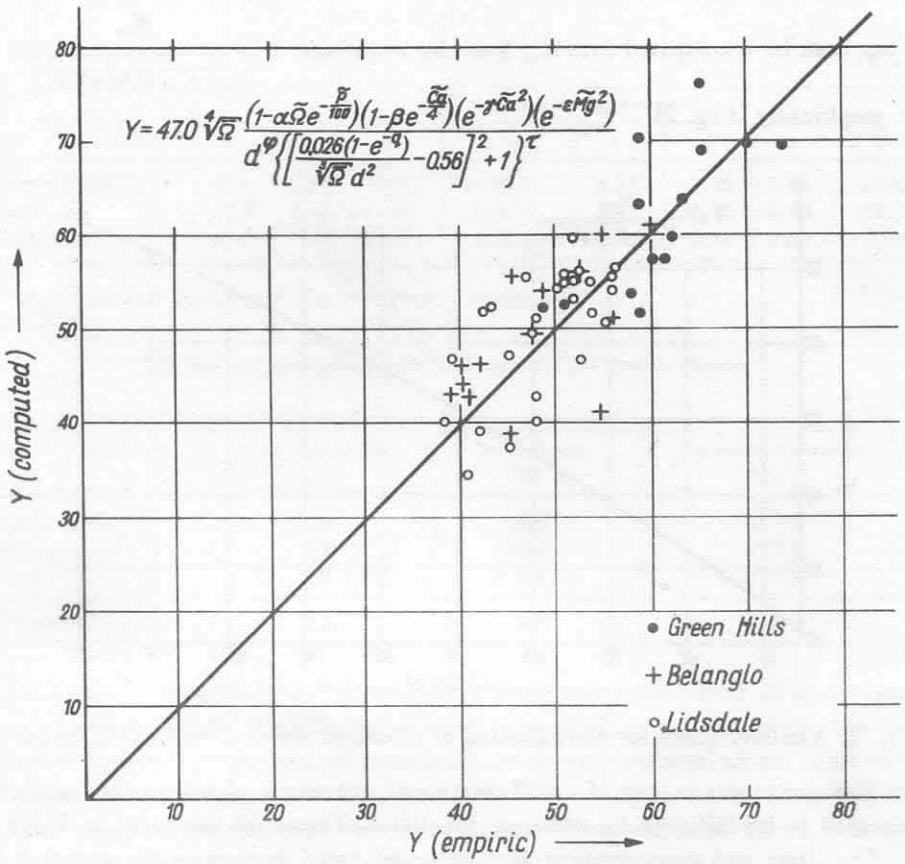

Fig. 1. Comparison between empirically obtained values of Y and computed ones by equation (10) (the values of Y can be recomputed into site indices (H_{20}) by formula (9)) $\tilde{\Omega}$ – effective upright-soil ratio, $\tilde{P}$ – effective P in ppm, $\tilde{Ca}$ – effective Ca in me/100 g, $\tilde{Mg}$ – effective Mg in me/100 g, d – mean size of soil particles, q – humidity index, e – base of natural logarithms, α, β, γ, ε, φ, τ – constant values for particular localities, depending on climate conditions (see Tab. III)

author, to combine the Mitscherlich and Czarnowski equations into an equation presented in Figure 1.

In place of the height of the stand at the age of 20 years (H_{20}) as the dependent variable, the expression used is:

$$Y \approx \frac{H_{20}}{\sqrt{1+\gamma_2 \cdot H_{20}}} \tag{9}$$

from which it can be estimated very provisionally that with H_{20} in feet:

$$\gamma_2 = 0.0127$$

Y can then be recomputed into H_{20} from the equation: $Y = \dfrac{H_{20}}{\sqrt{1 + 0.0127\, H_{20}}}$

or graphically (Fig. 2).

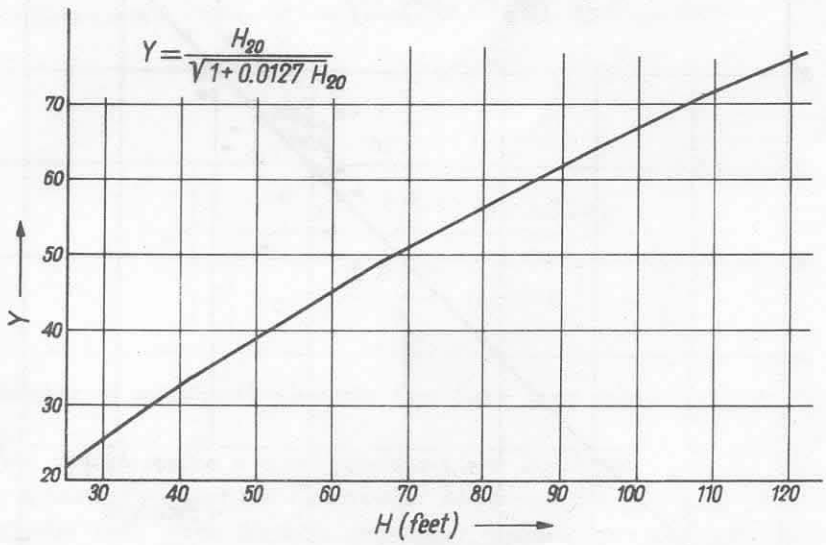

Fig. 2. Auxiliary graph for determination of values of site-indices (H_{20}) by means of Y

The particular values of coefficients and exponents of independent variables appeared to be different for different localities. These are compared in Table III.

The first and most striking feature to be noted is that in the equation (10) the coefficient of proportionality (C) is the same for all localities. Secondly, the values of other coefficients of the equation (α, β), and the exponents (γ, φ) diminish when the climatic characteristic diminishes. These dependencies clearly indicate how strongly climatic factors are interrelated with soil chemical properties, and the steady trends allow us to make interpolations and even modest extrapolations to other localities.

The statistical reliability of the prediction equation is illustrated on Figure 1 and in Table IV.

In the aggregate the standard deviation is equal to ±10%. This is certainly not a very impressive standard of precision. Nevertheless so far as the present writers know, it is a very high level of precision for purposes of expressing site productivity quantitatively in terms of both climate and soil. From a purely pragmatic viewpoint such accuracy is sufficient to make predictions about probable success, or lack of it, in plantation schemes, with a measurable degree

Coefficients and exponents of independent variables of the equation (10) and in Figure 1

Tab. III

Locality	Coefficients and exponents*							
	C	α	β	γ	ε	φ	τ	Y
Green Hills	47.0	0.52	0.52	0.0027	0.02	0.42	0.35	67.0
Belanglo	47.0	0.46	0.40	0.0020	0.03	0.40	0.40	48.5
Lidsdale	47.0	0.40	0.29	0.0018	0.00	0.33	0.50	49.0

*Explanations of signs – see Figure 1 and the equation (10).

Statistical parameters of the equation (10) and in Figure 1

Tab. IV

Locality	No. of plots	Mean error (%)	Standard deviation (%)
Green Hills	14	±6.6	± 7.8
Belanglo	12	±10.2	±12.1
Lidsdale	27	± 9.5	±10.3
Total	53	± 8.9	±10.0

of certainty. It certainly places prediction well above the level of guess-work. By comparison, engineering formulas used in applied mechanics or statics (problems of facture and strength of materials), work with average errors up to ±15%. In engineering hydrology the best formulas have average errors no better than ±5% so that this initial trial on rather limited data, although drawn from very dissimilar plantations, can only be described as very promising.

The equation derived for the data is:

$$Y = 47\sqrt[4]{\tilde{\Omega}}\,\frac{\left(1-\alpha\tilde{\Omega}\,e^{-\frac{\tilde{P}}{100}}\right)\left(1-\beta\,e^{-\frac{\tilde{C}a}{4}}\right)\left(e^{-\gamma\tilde{C}a^2}\right)\left(e^{-\varepsilon\tilde{M}g}\right)}{d^{\varphi}\left\{\left[\frac{0.026\,(1-e^{-q})}{\sqrt[3]{\tilde{\Omega}}\;d^2}-0.56\right]^2+1\right\}^{\tau}} \tag{10}$$

It is not very surprising that its form is so complicated. Site productivity itself is the resultant of complex factors and this must inevitably be reflected in the structure of the equation. Objections can be raised about its obvious inelegance and artificiality. Such objections can be answered in part by stating that the natural and mathematical gist is clearly visible. This function is no doubt a mere simplification and a particular case statement of a more general and more legitimate algebraic function. It would be premature to impute to it at this

stage a more sophisticated form, but one possibility for transformation (based on the approximation,

$$\prod\left(1 - e^{-x^{\frac{1}{n}}}\right) = \left(1 - e^{-x^{\frac{1}{n}}}\right)^{n} \sim x, \tag{11}$$

valid, within certain limits, at low values of x) could have the form:

$$Y \sim \prod_{x=1}^{n} \left(1 - e^{-IC_x E_x \omega}\right)\left(e^{-b\, x^{E}\, x^{2\omega}}\right) \tag{12}$$

where:

$$I = \left\{ \frac{p'(1 - e^{-q})\,(1 - 0.01\,\iota)}{d\left\{ C_{\square}\left[C_4 \frac{(1 - e^{-q})}{d^2}\, f(\sigma) - W_f\right]^2 + 1\right\}} \right\}^{\frac{1}{n}} \tag{13}$$

n = number of nutritional elements that have been taken into consideration, $\omega \leqslant 1$.

This appears to be a more consistant and legitimate function. Thus there exists a possibility that the Czarnowski frame equation is a simplification and a particular case when the soil nutritional elements are not only low, but at the same time the humidity index, or the value of C is sufficiently low.

Similarly there exists the possibility that the Mitscherlich equation in its pure form finds its best application to fully fertilized soils, or even applies best to soil solution sources of nutrient rather than to poorer natural soils in which exchangeable nutrients are dominant sources.

The development of electronic computers, with their ability to handle complex computations rapidly, open up the possibility of solving the probit equation in a way which would not have been attempted previously because of the time needed, complexity of the phenomena, lack of a meaningful model, and lack of standardized, comparable data. The best computers and most talented statisticians will be powerless unless factors that play dominant roles in the phenomenon are all included.

An example is the soil porosity, or the closely related soil bulk density. The writers were not fully aware of the influence of soil porosity and productivity in sandy soils, but the equation (10) discussed here would indicate that there is a paradox involved in the influence of this characteristic on site productivity. That is to say, this factor still appears to have an important positive influence in a situation where its influence might seem more appropriately negative.

It is striking also that each of the components considered, with the exception of grossly deficient soil nutrients, can be said to be individually of little importance, but taken together, most of them are as important as the others. What becomes most evident is that the interrelationship between physical and chemical properties of the soil are such that omitting one characteristic brings about oversimplification to the point where total error becomes excessive.

In conclusion, the authors hope that readers will agree that further investigations of site productivity along empirical mathematical lines offer considerable promise. The planned and already-operating system of data collection within the International Biological Programme will certainly shed much more light on the limits restricting site productivity in man-made forests.

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WSKAŹNIK SIEDLISKOWY JAKO FUNKCJA CHARAKTERYSTYK GLEBOWYCH I KLIMATYCZNYCH (TYMCZASOWE DONIESIENIE O WYNIKACH BADAŃ UPRAW *PINUS RADIATA* D. DON. W NOWEJ POŁUDNIOWEJ WALII W AUSTRALII)

Streszczenie

Autorzy opracowali równanie (10) wyrażające wskaźnik siedliskowy (wysokość drzew dominujących w drzewostanie w wieku 20 lat) jako funkcję glebowych i klimatycznych charakterystyk, wykorzystując dane zebrane w standardowy sposób z trzech miejscowości w Nowej Południowej Walii (Australia). Przedmiotem studium są jednowiekowe i jednogatunkowe plantacje *Pinus radiata* D. Don.[1]. Opracowania dokonano stosując

[1] *Pinus radiata* D. Don. (synonim *Pinus insignis* Douglas) ma swoją ojczyznę na wybrzeżu Oceanu Spokojnego Ameryki Północnej w okolicy Monterey w Kalifornii, gdzie zajmuje nieznaczny areał swego naturalnego rozsiedlenia. Z wielkim powodzeniem gatunek ten uprawia się od lat kilkudziesięciu w Nowej Zelandii, Australii, Południowej Afryce i Południowej Ameryce. W Chile i Hiszpanii są nawet niemal nieznane inne egzoty w uprawach leśnych oprócz *Pinus radiata*. Tę niezwykłą karierę zawdzięcza ten gatunek znakomitym efektom hodowlanym. W Nowej Południowej Walii jego plantacje osiągają wysokość 35 m w wieku 20 lat. W Polsce nasza sosna pospolita, *Pinus silvestris* L., dochodzi do tej wysokości na najlepszych siedliskach w wieku 120 lat.

matematyczne metody, analogiczne do tych, jakie są w użyciu w hydrologii. Osiągnięto dokładność wyrażającą się współczynnikiem zmienności ±10,0%.

Symbole zastosowane w równaniu (10) oznaczają, co następuje:

Y – charakterystyka zdolności wytwórczej siedliska; funkcja wysokości drzewostanu osiąganej w wieku 20 lat, wyrażona równaniem (9), gdzie H_{20} oznacza tę wysokość.

$\widetilde{\Omega}$ – efektywna wartość udziału „rzetelnej gleby", tzn. stosunek objętościowy gleby o ziarnach mniejszych od 2 mm. Efektywną wartość obliczono wzorem (3), gdzie znacznik A oznacza horyzont glebowy 0–7,5 cm, zaś znacznik B – horyzont 30–40 cm.

e – podstawa logarytmów naturalnych.

$\widetilde{P}$ – efektywna wartość zawartości fosforu (całkowitego w ppm (tzn. w 1/1 000 000). Wartość tę obliczono wzorem (4), gdzie przez E oznaczono zawartość; znaczniki analogiczne jak pod $\widetilde{\Omega}$.

$\widetilde{Ca}$ – efektywna zawartość wapnia wymiennego w miliekwiwalentach/100 g.

$\widetilde{Mg}$ – efektywna zawartość magnezu.

d – średni rozmiar ziarn gleby w mm, obliczony metodą Kozenego (por. Czarnowski 1964).

q – wskaźnik wilgotności obliczony wzorem (5) (por. Czarnowski 1964, 1966).

α, β, γ, ε, φ, τ – stałe dla określonej miejscowości; ich wartości zestawiono w tabeli III.

Równanie (10) utworzono przy pomocy skojarzenia równania Mitscherlicha (zmodyfikowanego) oraz zrębowego równania Czarnowskiego (1964). Współczynniki (tab. III) zmiennych niezależnych wykazują konsekwentną zależność od wskaźnika wilgotności. Ta zależność umożliwia interpolację wartości wskaźnika siedliskowego dla innych miejscowości.

W obecnej fazie badań uzyskano równanie (10) w formie prowizorycznej, lecz jednocześnie zarysowuje się możliwość jego przekształcenia w równanie o konstrukcji podanej wzorem (12).

Zachodzi także możliwość, iż równanie Mitscherlicha (6), które było jednym z wątków wzoru (10), nie znajduje pełnego zastosowania w warunkach naturalnych gleb leśnych.

Rozstrzygnięcie wielu nasuwających się kwestii wymaga większej ilości danych empirycznych zebranych w ściśle standardowy sposób w możliwie najszerszej skali warunków glebowych i klimatycznych. *Pinus radiata* przedstawia do tego celu znakomity materiał badawczy z powodu rozpowszechnienia tego gatunku na wszystkich kontynentach. Międzynarodowy Program Biologiczny otwiera szerokie perspektywy zebrania niezbędnych danych w skali całego globu ziemskiego.

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